numerous perhaps than those which have been obtained directly. Holding in mind the great practical importance of a more complete understanding of the Sun and the high probability of light being thrown upon its problems by observations that can at present be made only during the brief and long-separated total eclipses, it is apparent that every opportunity should be embraced to make the most of these occasions. It is to the credit of the Lick Observatory that it has played a conspicuous part in the observation of eclipses of the Sun, and not too much can be said in commendation of the systematic giving by which Mr. WILLIAM H. CROCKER, and, before him, the late Colonel CHARLES F. CROCKER, have made possible the continuous study and investigation of these phenomena by that institution.

DETROIT OBSERVATORY, UNIVERSITY OF MICHIGAN,
ANN ARBOR, January 22, 1906.

NOTE ON ANOMALOUS REFRACTION.

BY FRANK SCHLESINGER AND G. B. BLAIR.

Under normal conditions atmospheric strata of uniform density lie parallel to the Earth's surface. In this case the expression for refraction takes the well-known form $k \tan z$, in which z is the true zenith-distance of the object and k is a quantity that varies slowly with the zenith-distance, the temperature, and the height of the barometer. For present purposes, however, we may regard k as a constant and equal to $57''$.

Let us now consider the effect upon the refraction of a small inclination in the strata of uniform density. Imagine a normal to be drawn to these strata, and let ζ and a be respectively the zenith-distance and azimuth of the point at which this normal pierces the celestial sphere. This point is evidently the origin from which zenith-distances should be reckoned for the computation of the refraction under the assumed conditions. Consequently for z in the above formula

we should substitute (with a sufficient degree of approximation) $z - \zeta \cos(a - A)$, A being the azimuth of the object.

Hence the correction arising from anomalous refraction is

$$k \tan [z - \zeta \cos(a - A)] - k \tan z$$

or very nearly $$k \cdot \zeta \cos(a - A) \sec^2 z \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

For objects in the meridian this becomes

$$\pm k \zeta \cos a \sec^2 z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The positive or negative sign is to be used according as the object is south or north of the zenith.

The following table gives the values for various zenith-distances of the coefficient of $\zeta \cos a$, the latter being expressed in minutes of arc:—

TABLE I.

z	Anomalous Refraction.
0°	0".017 $\zeta \cos a$
10	0 .017 "
20	0 .019 "
30	0 .022 "
40	0 .028 "
50	0. 040 "
60	0 .067 "
70	0 .142 "

This table shows that the effect of anomalous refraction is nearly the same for all objects that culminate within 30° of the zenith, but that it increases rapidiy when the zenith-distance surpasses 50°.

It will be instructive to apply the above formula to actual observations. So far as we know, the only ones suitable for the purpose are those which have been made at the six international latitude stations. The programme for observing at these stations includes, in each night's work, twelve pairs at small zenith-distances (never more than 23°), and four pairs at large zenith-distances[1] (about 60°). An ordinary programme—that is, one which contains pairs at small zenith-distances only—cannot be used for our purpose, since, as we have seen from Table I, the effect would be nearly the

[1] For brevity we shall refer to these as refraction pairs and to those near the zenith as zenith pairs.

same for all the pairs, and it would be impracticable to distinguish a refraction effect from any other disturbing phenomenon. Furthermore, it would be equally futile to attempt to deduce the value of $\zeta \cos \alpha$ from absolute measures of declinations, since these do not possess the requisite accuracy.

The results of the first two years of the international work have been published by Dr. ALBRECHT in volume I of the "Resultate des Internationalen Breitendienstes." On the data given in this memoir Table II is based.

Column 1 gives the name of the station and column 2 the number (n) of nights during 1900 and 1901 on which all four of the refraction pairs, and at least ten of the twelve zenith pairs, were observed. No use is made of the other nights in the present paper, which, even with these restrictions, covers more than fourteen thousand separate determinations of the latitude. Column 3 contains the sums of the squares of the "zenith divergences" obtained thus: ALBRECHT'S Plate XI shows with red lines the definitive latitudes at the six stations; and on pages 130 to 139 are given the results for each night from the zenith pairs. The difference between these two quantities was taken for each night, and the sum of their squares for the whole two years is entered in column 3.

TABLE II.

1	2 n	3 Zenith.	4 Refract.	5 Zenith minus Refract.	6 $\frac{\epsilon^2}{4} + \rho^2$
Mizusawa	77	0.2797	1.7880	1.6148	0.00073
Tschardjui	131	1.0221	3.4920	4.2131	0.00029
Carloforte	258	0.7848	3.6875	3.7293	0.00036
Gaithersburg ...	169	0.6422	2.6526	2.4720	0.00061
Cincinnati	131	0.6407	3.4664	3.0565	0.00100
Ukiah	165	0.3737	3.8113	4.1683	0.00001
Means	155				0.00050

Let ϵ be the mean error of the definitive latitude shown in ALBRECHT'S Plate XI.

ϵ_z, the mean error of the mean of the ten to twelve zenith pairs observed each night.

ρ, the mean effect of anomalous refraction at the zenith.

Then the numbers in column 3 may be represented by the expression $\Sigma\,(\epsilon^2 + \epsilon_z^2 + \rho^2)$ . . . . (3)

Column 4 contains quantities similar to those in column 3, except that the refraction pairs (pages 143 to 151) have been used instead of those near the zenith. Representing by ϵ_r the mean error of the mean of the four refraction pairs, and remembering that at 60° zenith-distance the effect of anomalous refraction is four times that at the zenith, the numbers in column 4 are the equivalents of

$$\Sigma\,(\epsilon^2 + \epsilon_r^2 + 16\rho^2) \quad \ldots\ldots \quad (4)$$

Column 5 gives the values of

$$\Sigma\,(\epsilon_z^2 + \epsilon_r^2 + 9\rho^2) \quad \ldots\ldots \quad (5)$$

obtained by taking the differences between the refraction pairs for the same night, squaring and adding.

By adding corresponding items in columns 3 and 4, subtracting those in column 5, and dividing by $8n$, we evidently obtain $\frac{\epsilon^2}{4} + \rho^2$ . . . . (6)

the values of which are given in column 6.

The data in Table II do not permit us to separate these two errors, but from other considerations[1] it is known that ϵ is about $\pm$ 0″.02. Whether we assume this value of ϵ or a smaller one, we should get practically the same values of ρ, as is shown by the following:—

TABLE III.

Values of ρ, assuming:	$\epsilon = \pm$ 0″.02	$\epsilon =$ 0″.00
Mizusawa	$\pm$ 0″.025	$\pm$ 0″.027
Tschardjui	.014	.017
Carloforte	.016	.019
Gaithersburg	.023	.025
Cincinnati	.030	.031
Ukiah	.000	.003
Means	$\pm$ 0″.018	$\pm$ 0″.020

Adopting the former as definitive, and substituting in expression (3) above, we get these values for the mean

[1] See the residuals on pages 163 to 166 of ALBRECHT'S memoir.

errors of the latitude as determined from one night's observations:—

TABLE IV.

	Values of ϵ_z.
Mizusawa	$\pm 0''.051$
Tschardjui	.085
Carloforte	.049
Gaithersburg	.054
Cincinnati	.060
Ukiah	.044
Mean	$\pm 0''.057$

The conclusion that we may draw from these computations is that observers have little to fear from anomalous refraction. Its mean effect, at a properly chosen station, appears to be considerably less than the accidental errors of observation in the best work that can be done at the present time. Accordingly, this explanation for inconsistencies in meridian work should be advanced with caution. The present paper also throws light upon the nature of KIMURA's phenomenon; this, it will be remembered, is a small term in the latitude variation and is independent of the longitude. Our computations indicate, if they do not indeed prove, that this term is real, and is not due (as has been suggested) to anomalous refraction.

ALLEGHENY OBSERVATORY, December 22, 1905.

VARIABLE STAR NOTES.

BY ROSE O'HALLORAN.

U Cassiopeiæ.

On September 17, 24, and October 15, 1905, this variable was invisible in a four-inch telescope. The maximum predicted for November 3d was looked for, and according to the following observations occurred somewhat later.

1905. October 23, 26—Of 12th magnitude. November 3—About 11.7; dimmer than *g*.[1] November 13—Between *g* and

[1] For comparison-stars, see chart, *Publications* A. S. P. No. 98, p. 209.